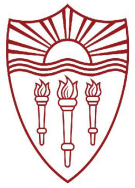


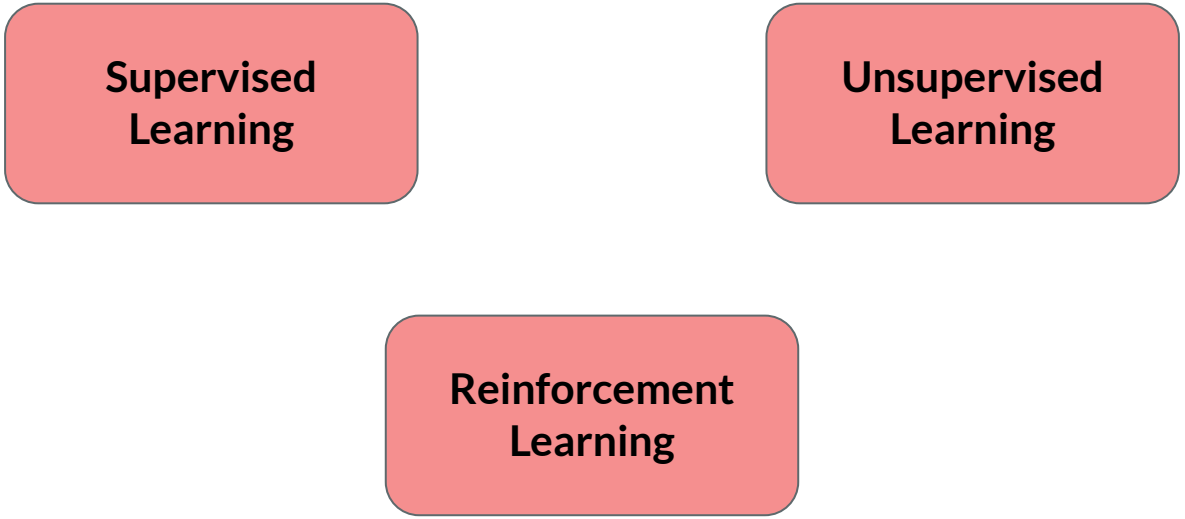
CSCI 467: Machine Learning

Discussion – Review of Linear Methods

Slides by: Ameya Godbole (Feb 2024)
Materials from: https://usc-csci467.github.io/

High level categories in ML

Based on objectives



Parametric vs Non-Parametric

Learn a **fixed-size** set of parameters	Learnable parameters depends on size of training data
Can throw away data after training	Predictions uses training data

Despite the name, non-parametric **does not mean** “no parameters”

Simplest Example: k-Nearest Neighbors

- On test example x, predict the **most common label** among the **k neighbors of x**
- Nearby examples have the same label
- Need to define a **similarity function**

- **Low bias:** Makes no assumption about decision boundary
- **Potentially high variance:** Decisions are very sensitive to training data
  - “Curse of dimensionality”

Complex Example: Kernel Methods

- Like k-NN, decision depends on “nearby” training data
- **Kernel functions:** (loosely) **measures closeness** of two points

$$f(x) = \sum_{i=1}^{n} \alpha_i k(x^{(i)}, x^{\text{test}})$$

- α ‘s are parameters: one for each training data point
- How do we learn them?
  - Lec 8 covered connection between logistic regression and an algorithm for learning α
  - SVMs

Kernel Trick

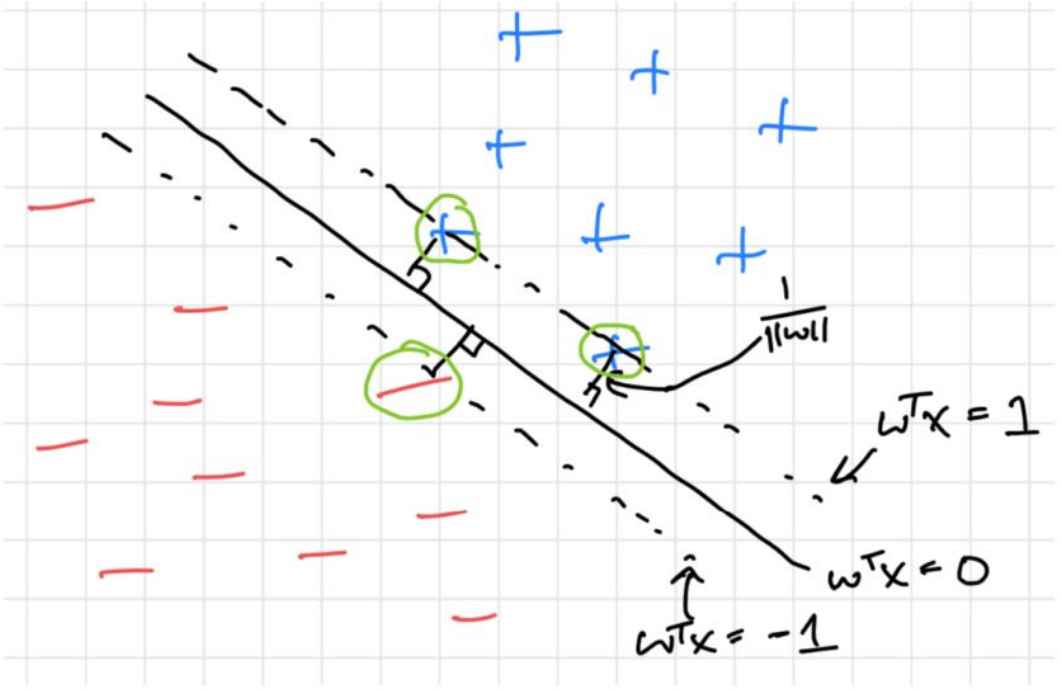
Use kernel functions between two data points
instead of
Computing dot product in large feature spaces

Why use kernel methods?

+ Get benefit of using large feature spaces (more expressive)
+ Avoid computation of explicitly writing out the data point in a large feature space
    - RBF kernels represent dot product in infinite dimensional space
    - You'll work through this in the homework
- Inference time has a dependence on number of training datapoints

Support Vector Machines

- Max-margin classifier
  - Objective function incentivizes a large margin between the two class

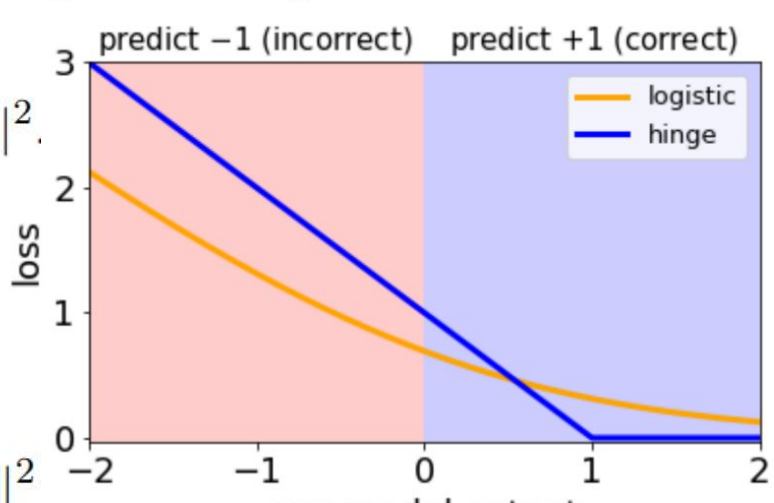


SVM

- Can be derived as a special case of replacing logistic loss with hinge loss

$$L(w) = \frac{1}{n}\sum_{i=1}^{n} -\log \sigma(y^{(i)} \cdot w^\top x^{(i)}) + \lambda \|w\|^2$$

$$L(w) = \left(\frac{1}{n}\sum_{i=1}^{n} [1 - y^{(i)} w^\top x^{(i)}]_+\right) + \lambda \|w\|^2$$



SVM

- Can then be kernelized to take advantage of kernel trick

$$L(\alpha) = \frac{1}{n}\sum_{i=1}^{n}\left[1 - y^{(i)}\sum_{j=1}^{n}\alpha_j k(x^{(j)}, x^{(i)})\right]_+ + \lambda\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j k(x^{(i)}, x^{(j)})\right)$$

SVM: What are support vectors?

- Like other kernel methods, the SVM decision function can be stated as:

$$f(x) = \sum_{i=1}^{n} \alpha_i k(x^{(i)}, x^{\text{test}})$$

- However, optimizing the SVM objective leads to:
  - α being non-zero only for **support vectors:** examples that lie on the decision boundary or within the margin and misclassified examples
  - α being zero for examples that are correctly classified
- This means at test time, the decision depends only on a small number of support vectors and not the entire training set

SVM: Optimization

- There exist alternative techniques to minimize SVM loss
  - Linear optimization libraries
- These approaches only require dot-products between the data points
  - Thus, they can use kernel functions

- Combine this with efficient prediction using kernels

This is the big reason why SVMs and kernels are a good match!

Quick Note:
All non-parametric approaches discussed, have corresponding variants for regression