$$P(A=b \mid B=b) = \frac{P(B=b \mid A=b)\cdot P(A=b)}{\underbrace{P(B=b)}}$$

$$= \frac{P(B=b \mid A=b)\cdot P(A=b)}{P(B=b \mid A=b)\cdot P(A=b) + P(B=b \mid A=r)\cdot P(A=r)}$$

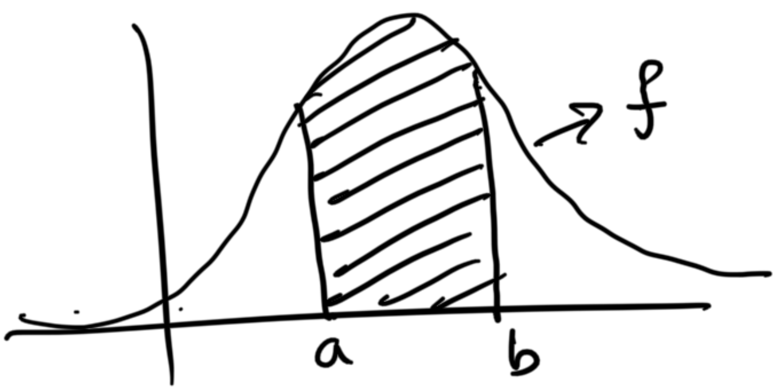
2R
3B

$$= \frac{\left(\frac{2}{4}\right)\cdot\left(\frac{3}{5}\right)}{\left(\frac{2}{4}\right)\cdot\left(\frac{3}{5}\right) + \left(\frac{3}{4}\right)\left(\frac{2}{5}\right)}$$

$$= \frac{6}{6+6} = \frac{1}{2}$$

$P(A=r)$ → Prob. density → PMF ~~po~~ prob mass f^n (discrete events)

→ cont variable → PDF prob. density f^n.



$$P(a \le X \le b) = \int_a^b f(x)\,dx$$

$$\left.\begin{aligned} P(A=r) &= 2/5 \\ P(A=b) &= 3/5 \end{aligned}\right\}$$ PDF of A picking a ball

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① $f(x) = e^x$

input $x \in \mathbb{R}$
output $f(x) \in \mathbb{R}$

$$\frac{d f(x)}{dx} = e^x$$

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② $f(\bar{x}) = 3x^2 y$

$\bar{x} \in \mathbb{R}^n$ $n=2$
$f(x) \in \mathbb{R}$

---

③ $\bar{f}(x) = \begin{bmatrix} 2x \\ \end{bmatrix}$

$x \in \mathbb{R}'$
$\bar{f}(x) \in \mathbb{R}^n$ $n=2$

③ $f(x) = \begin{bmatrix} 3x^2 \end{bmatrix}$ $\quad f(x) \in \mathbb{R}$

④ $\bar{f}(\bar{x}) = \begin{bmatrix} 3x^2 y \\ 2x + 5y \\ 4xy \end{bmatrix}$

$\bar{x} \in \mathbb{R}^n \qquad n = 2$

$\bar{f}(x) \in \mathbb{R}^m \qquad m = 3$

$$\bar{f}(\bar{x}) = \begin{bmatrix} f_1(\bar{x}) \\ f_2(\bar{x}) \\ \vdots \\ f_m(\bar{x}) \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$\bar{f} \in \mathbb{R}^m$

$\bar{x} \in \mathbb{R}^n$

$$\frac{\partial \bar{f}(\bar{x})}{\partial \bar{x}} = \begin{bmatrix} \frac{\partial}{\partial \bar{x}} f_1(\bar{x}) \\ \frac{\partial}{\partial \bar{x}} f_2(\bar{x}) \\ \vdots \\ \frac{\partial}{\partial \bar{x}} f_m(\bar{x}) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial}{\partial x_1} f_1(\bar{x}) & \frac{\partial}{\partial x_2} f_1(\bar{x}) & \cdots & \frac{\partial}{\partial x_n} f_1(\bar{x}) \\ \vdots & \ddots & & \vdots \\ \frac{\partial}{\partial x_1} f_m(\bar{x}) & \cdots & \cdots & \frac{\partial}{\partial x_n} f_m(\bar{x}) \end{bmatrix}$$

[Jacobian Matrix]

Numerator Layout of Vector Gradients

	$1\ \boxed{x}$	$n\ \boxed{\bar{x}}$
$1\ \boxed{f}$	$1\ \boxed{\frac{df}{dx}}$	$1\ \boxed{df/d\bar{x}}$ (n)
		n

$m\ \bar{f}$ | $m\ \frac{d\bar{f}}{dx}$ | $m\ \frac{d\bar{f}}{d\bar{x}}$ → Jacobian

$\bar{y} = A\bar{x}$ $\quad \bar{y} \in \mathbb{R}^m \quad \bar{x} \in \mathbb{R}^n$

$A \in \mathbb{R}^{m \times n}$

$$\frac{d\bar{y}}{d\bar{x}} = A$$

$\alpha = y^T A x \quad y \in \mathbb{R}^{m \times 1} \quad x \in \mathbb{R}^{n \times 1}$

$A \in \mathbb{R}^{m \times n} \quad \alpha \in \mathbb{R}$

$$\frac{\partial \alpha}{\partial \bar{x}} = y^T A$$

n

$$\frac{\partial \alpha}{\partial \bar{y}} = (Ax)^T = x^T A^T$$

m $\quad (1 \times m)$

$(1 \times m)(m \times n) = (1 \times n)$

$(m \times n)(n \times 1) = (m \times 1)^T$

$\alpha^T = x^T A^T y$

$$\frac{\partial \alpha^T}{\partial y} = x^T A^T$$

$\alpha = x^T A x$

$$\frac{\partial \alpha}{\partial \bar{x}} = x^T(A + A^T)$$

n