USC CSCI 467 Intro to Machine Learning	**Final Exam** May 7, 2024, 2:00-4:00pm	Spring 2024 Instructor: Robin Jia

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Answer the questions in the spaces provided. **If you run out of space, continue your work on the last two pages, and indicate that your answer is there**. You may use the backs of pages for scratch work only. **Please use pen for ease of grading**. This exam has 6 questions, for a total of 150 points.

Question 1: Language Model-Based Generative Classifier (25 points)

Lorena wants to analyze social media posts about the upcoming election. Therefore, she wants to build a classifier that takes in a post, and classifies it either as having a liberal (L), conservative (C), or neutral (N) perspective. However, she does not have any training data for this task.

Lorena has an idea: she can use a *pre-trained* Transformer language model to help. She writes three *prompts*, one for each label:

- Liberal prompt s_L: "This is a liberal-leaning social media post:"
- Conservative prompt s_C: "This is a conservative-leaning social media post:"
- Neutral prompt s_N: "This is a politically neutral social media post:"

To make a classification decision on a post x, she can ask the language model to predict the probability of generating post x after each one of these prompts. For instance, if x is the string "I support Joe Biden," she would measure the probability of generating "I support Joe Biden" after each of the three prompts.

More formally, she computes $p(x \mid s_L)$, $p(x \mid s_C)$, and $p(x \mid s_N)$ using the language model. She then treats these values as the probability of x given that the label y is either liberal, conservative, or neutral, respectively.

Finally, she decides to assume that all three labels are equally likely. This allows her to build a *generative* classifier. Recall that given a test input x, a generative classifier estimates $P(y)$ and $P(x \mid y)$ for each possible label y, and then uses Bayes Rule to make a prediction.

(a) (2 points) Let x be a post consisting of just two words x_1 and x_2 (in that order). Which of the following accurately describes the process of computing $p(x \mid s_L)$? For two strings u and v, let $u + v$ denote the concatenation of those strings. Circle the best answer.

A. Multiply $p(x_1 \mid s_L)$ and $p(x_2 \mid s_L)$
B. Multiply $p(x_1 \mid s_L)$ and $p(x_2 \mid s_L + x_1)$
C. Add $p(x_1 \mid s_L)$ and $p(x_2 \mid s_L)$
D. Add $p(x_1 \mid s_L)$ and $p(x_2 \mid s_L + x_1)$

(b) (5 points) Using the notation from the question preamble, write the formula for $P(y = C \mid x)$, the probability that a post x is conservative according to Lorena's classifier.

(c) Lorena remembers that you should always inspect your data when doing machine learning. After reading many social media posts, she realizes that her previous assumption that all three labels are equally likely is wrong. Actually, liberal and conservative posts each occur around 45% of the time, and only 10% of posts are neutral.

i. (3 points) Write the modified formula for $P(y = C \mid x)$, taking into account this information.

ii. (5 points) Will Lorena need to re-train her language model to account for this? Explain why or why not.

(d) Lorena gets tired of looking at social media posts, and instead wants to analyze bills that were proposed in Congress. She again wants to identify whether they promote liberal, conservative, or neutral policies. These bills are much longer than social media posts. Lorena is considering whether she should switch from a Transformer language model to an RNN language model to analyze these much longer documents.

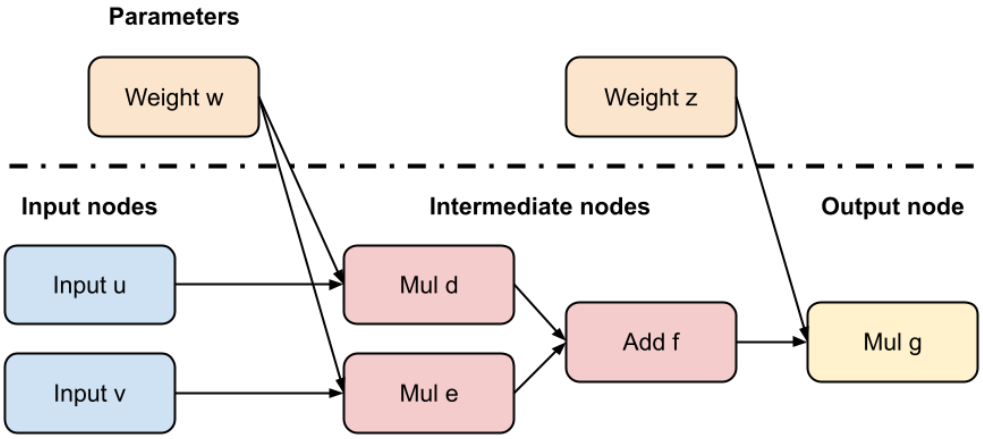
i. (5 points) Give one reason why switching to an RNN would be a **good** idea. Explain your reasoning.

ii. (5 points) Give one reason why switching to an RNN would be a **bad** idea. Explain your reasoning.

Question 2: Backpropagation (31 points)

In this question, we are going to work through the computation of gradients for a simplified neural network. Our focus will be on handling gradients for shared parameters. These are parameters that are used multiple times in the forward pass of the network.

Consider the simplified neural network depicted below. Let's work through the implementation of backpropagation for the network. The network consists of input nodes u and v and parameter nodes w and z, which are all scalar numbers for this question. Intermediate computations are named d, e, f. Notice that w is a shared parameter that is used twice in the computation of g.



$$d = w \times u$$
$$e = w \times v$$
$$f = d + e$$
$$g = z \times f$$

(a) (3 points) The first thing we need to do is maintain a topological ordering of the nodes. This list will be used (in the reverse order) to compute gradients during backpropagation. Complete this topologically sorted list:

$$u, v, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, g$$

(b) Now, compute the partial derivatives of g with respect to other nodes in the graph. Remember that you can reuse gradient expressions that you have previously computed, just like in backpropagation. Show your work. We have copied over the formulas from the previous page for convenience:

$$d = w \times u$$
$$e = w \times v$$
$$f = d + e$$
$$g = z \times f$$

(c) (2 points) Compute $\frac{\delta g}{\delta z}$.

(d) (2 points) Compute $\frac{\delta g}{\delta f}$.

(e) (3 points) Compute $\frac{\delta g}{\delta d}$.

(f) (3 points) Compute $\frac{\delta g}{\delta e}$.

(g) (4 points) Compute $\frac{\delta g}{\delta w}$.

(h) (8 points) Let's bring it all together. Suppose we are given the following loss function:

$$\mathcal{L} = \sum_{i=1}^{N} (y^{(i)} - g^{(i)})^2 = \sum_{i=1}^{N} (y^{(i)} - NN(u^{(i)}, v^{(i)}))^2,$$

where $g^{(i)} = NN(u^{(i)}, v^{(i)})$ is the output of applying the neural network on the features $(u^{(i)}, v^{(i)})$ of datapoint i and $y^{(i)}$ is the scalar target output for datapoint i. Compute $\frac{\delta \mathcal{L}}{\delta w}$. Show your work.

(i) (6 points) In this architecture, w is a shared parameter, as it is used in multiple independent computations. Name two architectures from class that use shared parameters, and state which parameters are shared.

Question 3: Principal Component Analysis (23 points)

Consider a dataset $\mathbf{X}$ consisting of the following four data points in a two-dimensional space:

$$\mathbf{X} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 5 & 4 \\ 1 & 0 \end{bmatrix}$$

We want to represent the data in only one-dimension and turn to principal components analysis (PCA) for this.

(a) To run PCA, there is an important preprocessing step that must first be done to $\mathbf{X}$.

i. (3 points) In English, describe what this step is.

ii. (3 points) Compute $\tilde{\mathbf{X}}$, the result of applying this preprocessing step to $\mathbf{X}$.

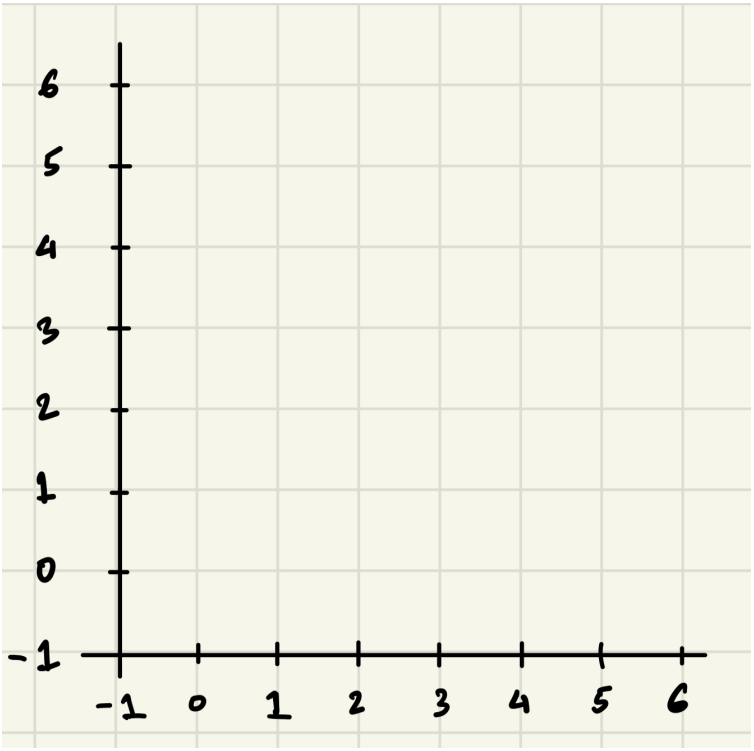
(b) (4 points) Calculate the covariance matrix $\mathbf{\Sigma}$, using your $\tilde{\mathbf{X}}$ computed in the previous part.

(c) If the answer to your previous part is correct, the eigenvectors of $\mathbf{\Sigma}$ are $\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}^\top$ with eigenvalue 1 and $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^\top$ with eigenvalue 4.

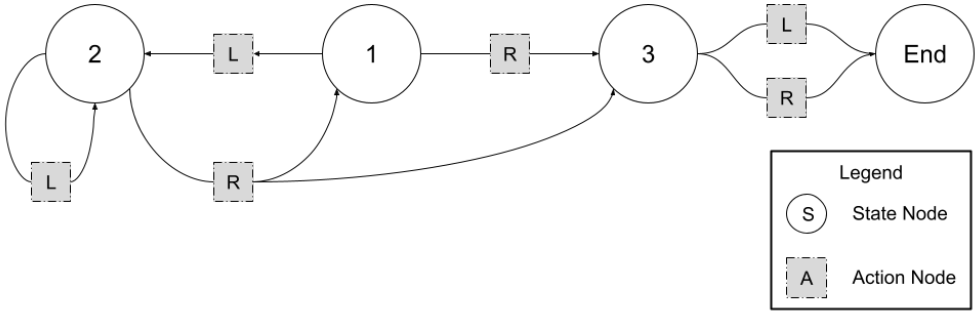
i. (4 points) By running PCA, you can find the one-dimensional subspace that best fits the shape of your data. Based on the eigenvectors and eigenvalues above, this subspace is spanned by what vector? How do you know?

ii. (3 points) Compute the result of projecting the four datapoints onto one-dimensional space.

(d) (6 points) Plot the **original** data points, the principal component direction (as a line), and the projections of all four sample points onto the principal direction. Label each projected point with its principal coordinate value.



Question 4: Reinforcement Learning (29 points)



Consider the simple Markov Decision Process (MDP) in the above figure. It consists of the 3 normal states `1`, `2`, `3` and the special `End` state. An agent starts in State `1` and roams around this MDP with two actions: move left '`L`' and move right '`R`'. The action nodes represent the chance nodes that represent the outcomes of actions (these are not under our control).

Consider that we know the following about the MDP:

- State Transition Function $\mathbf{T}$:
  - $\mathbf{T}(1, L, 2) = 1$
  - $\mathbf{T}(1, R, 3) = 1$
  - $\mathbf{T}(2, L, 2) = 1$
  - $\mathbf{T}(2, R, 1) = 0.5$
  - $\mathbf{T}(2, R, 3) = 0.5$
  - $\mathbf{T}(3, L, End) = 1$
  - $\mathbf{T}(3, R, End) = 1$
- Reward Function $\mathbf{R}$:
  - $\mathbf{R}(1, L, 2) = 5$
  - $\mathbf{R}(1, R, 3) = 10$
  - Rewards for all other transitions (s, a, s') are 0

(a) (2 points) What is $\mathbf{T}(1, R, 2)$?

(b) Ameya is running tabular Q-learning on this MDP. So far, he has learned the following Q-value estimates:

$\hat{Q}(s,a)$	Action L	Action R
State 1	15	10
State 2	4	8
State 3	0	0

The Q-learning agent is now about to select the next action to perform. The agent is currently in state 2. Ameya is using ε-Greedy with an ε value of 0.2.

i. (3 points) What is the probability that the agent chooses the R action?

ii. (3 points) What is the probability that the agent lands in state 1 at the next timestep?

iii. (5 points) During Q-Learning, does the learner have enough information to compute the answer to part (i)? Explain your answer.

iv. (5 points) During Q-Learning, does the learner have enough information to compute the answer to part (ii)? Explain your answer.

v. (6 points) Finally, suppose that the agent does the action R and transitions to state 1. How should the Q-value table be updated? Specify which cell(s) need to be updated, and what new value they will have. Use a discount factor of $\gamma = 0.8$ and learning rate of $\eta = 0.25$.

(c) (5 points) For this same scenario, Ameya wants to train a reinforcement learning agent that obtains higher expected reward than tabular Q-Learning. He has the idea to define a policy $\pi(s_{t-1}, s_t)$ which is a function of the current and the past state. Do you think this policy would obtain a higher expected reward? Explain your answer.

Question 5: Short Response (12 points)

Answer the following questions and **explain your reasoning fully**. You may also draw explanatory diagrams when appropriate.

(a) (6 points) Consider an attention head in a multi-headed attention layer within a Transformer encoder model. The input is a sequence of vectors $x_1, \ldots, x_T$. Your friend reasons that because x_T is more similar to itself than to any other vector, the attention from x_T will always attend more to x_T than to any other vector. Based on the mathematical formula for multi-headed attention, explain why your friend's reasoning is incorrect.

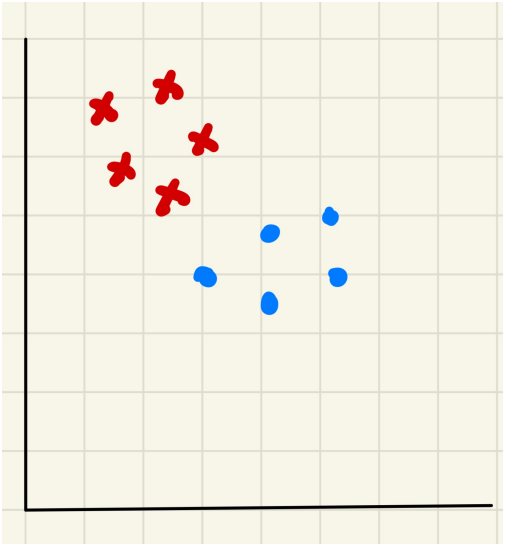
(b) (6 points) The first problem in the midterm exam involved a real-life machine learning application of estimating the current weight of a fetus based on ultrasound measurements of the baby's width/height. You might be wondering, how did they collect training data for this model? Researchers collected data on various babies' weights when they were born, and paired this with ultrasound measurements taken right before they were born. To get babies who were in a variety of developmental stages, the researchers included many babies who were born prematurely. Identify and describe a spurious correlation in this data that could bias the model's predictions.

Question 6: Multiple Choice (30 points)

In the following questions, circle the correct answer(s). There is no need to explain your answer.

(a) (3 points) The picture below shows a set of points with two clusters in crosses and circles. Which algorithm(s) could have generated these cluster assignments?

Choose all that apply.



A. Gaussian Mixture Models

B. k-Means

C. k-Nearest Neighbors

D. Principal Components Analysis

(b) (3 points) Wenyang is learning a discriminative model with parameters θ and wants to apply the principle of maximum likelihood estimation. She has a dataset of examples $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(n)}, y^{(n)})\}$. Which of the following describes what she should do? Choose all that apply.

A. Maximize $\log\left(\sum_{i=1}^{n} p(y^{(i)} \mid x^{(i)}; \theta)\right)$

B. Maximize $\sum_{i=1}^{n} \log p(y^{(i)} \mid x^{(i)}; \theta)$

C. Maximize $\prod_{i=1}^{n} p(y^{(i)} \mid x^{(i)}; \theta)$

D. Minimize $\sum_{i=1}^{n} \log p(y^{(i)} \mid x^{(i)}; \theta)$

(c) (3 points) Which of the following would cause running stochastic gradient descent to be equivalent to normal gradient descent? Choose all that apply.

A. Setting the batch size to the size of the training dataset.

B. Setting the batch size of 1.

C. Setting the number of epochs to the size of the training dataset.

D. Setting the number of epochs to 1.

(d) (3 points) Which of the following conditions could be a sensible stopping condition while building a decision tree?

A. Stop if you find the validation error is decreasing as the tree grows

B. Don't split a tree node that has an equal number of sample points from each class

C. Don't split a tree node whose depth exceeds a specified threshold
D. Don't split a tree node if the split would cause a large reduction in the weighted average

(e) (3 points) For which of the following is gradient descent is used for optimization? Choose all that apply.
A. Normal equations for Linear Regression
B. Feed-forward Neural Networks
C. k-NN Classifier
D. Gaussian Mixture Models

(f) For each of the statements below, indicate if they are true for:
A. Bandits only
B. Full reinforcement learning only
C. Both bandits and full reinforcement learning
D. Neither bandits nor reinforcement learning

i. (2 points) The agent's actions can influence what data is observed by the learning algorithm.

i. _____

ii. (2 points) The agent's action in the current timestep can influence which action is optimal in the next timestep.

ii. _____

iii. (2 points) If the agent takes the same action at timestep 1 and timestep 2, the expected reward received immediately after taking the action is the same.

iii. _____

(g) Soumya has a dataset of images $x_1, x_2, \ldots, x_n$. For each image i, he has a label y_i that says if the image is a cat or a dog. In each scenario, which of the following methods would be appropriate? Choose all that apply.
A. Expectation-Maximization with Gaussian Mixture Models
B. k-Nearest Neighbors
C. Linear Regression
D. Logistic Regression
E. Policy Gradient
F. Upper Confidence Bound Algorithm

i. (3 points) Soumya wants to find out whether there are distinct subtypes of cats within his group of cat pictures.

i. _____

ii. (3 points) Soumya wants to determine whether a new image x is a cat or a dog.

ii. _____

iii. (3 points) Given a new image x, Soumya wants to estimate the posterior probability of x being a cat.

iii. _____

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