$$f(x) = \frac{1}{1+e^{-x}} \rightarrow \text{Sigmoid}$$

↳ Logistic Regression

$$\frac{d(f(x))}{dx} = ?$$

Sol^n:

$$\frac{d}{dx}\left(\frac{U(x)}{V(x)}\right) = \frac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{[v(x)]^2} \quad (1)$$

$$f(x) = \frac{e^x}{e^x + 1}$$

$$\boxed{\frac{d}{dx}(e^x) = e^x}$$

$$\frac{d}{dx}(f(x)) = \frac{(e^x+1)(e^x) - e^x(e^x)}{(e^x+1)^2}$$

$$= \frac{e^x}{(e^x+1)^2} = \boxed{\frac{e^x}{(e^x+1)}} \cdot \frac{1}{(e^x+1)}$$

$$= f(x) \cdot \frac{1}{(e^x+1)}$$

$$\boxed{\frac{d(f(x))}{dx} = f(x) \cdot (1 - f(x))}$$

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(1) $f(x) = e^x$ input $= x \in \mathbb{R}$, output $= f(x) \in \mathbb{R}$

(2) $f(\bar{x}) = 3x^2y$ $\bar{x} \in \mathbb{R}^n$, $f(\bar{x}) \in \mathbb{R}$ $\quad \bar{x} = (x, y)$, $n=2$

$x \in \mathbb{R}$

③ $$\bar{f}(x) = \begin{bmatrix} 2x \\ 3x^2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad \bar{f}(x) \in \mathbb{R}^n \qquad n=2$$

④ $$\bar{f}(\bar{x}) = \begin{bmatrix} 3x^2 y \\ 2x+5y \\ \vdots \end{bmatrix} \qquad \bar{x} = (x, y), \quad \bar{x} \in \mathbb{R}^n, \quad \bar{f}(\bar{x}) \in \mathbb{R}^n$$

$$\boxed{y = \underline{W}\,\underline{x} + \underline{b}}$$

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② $f(\bar{x}) \in \mathbb{R}$ $\qquad \bar{x} \in \mathbb{R}^n$, $\quad \bar{x} = [x_1, x_2, \dots, x_n]^T$

$$\frac{d}{dx}(f(\bar{x})) = \left[\frac{\partial}{\partial x_1} f(\bar{x}) \quad \frac{\partial}{\partial x_2} f(\bar{x}) \quad \dots \quad \frac{\partial}{\partial x_n} f(x) \right]$$

④ $\bar{f}(\bar{x}) \in \mathbb{R}^m \qquad \bar{x} \in \mathbb{R}^n$

$$\frac{d}{dx}(\bar{f}(\bar{x})) = \begin{bmatrix} \frac{d}{dx}[f_1(\bar{x})] \\ \frac{d}{dx}[f_2(\bar{x})] \\ \vdots \\ \frac{d}{dx}[f_m(\bar{x})] \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial}{\partial x_1} f_1(\bar{x}) & \frac{\partial}{\partial x_2} f_1(\bar{x}) & \dots & \frac{\partial}{\partial x_n} f_1(\bar{x}) \\ \frac{\partial}{\partial x_1} f_2(\bar{x}) & & & \vdots \\ \vdots & & & \frac{\partial}{\partial x_n} f_m(\bar{x}) \\ \frac{\partial}{\partial x_1} f_m(\bar{x}) & \dots & & \end{bmatrix}$$

$\frac{\partial}{\partial x_1} f_m(\cdot)$

Jacobian Matrix

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} \qquad f'(x) = \begin{bmatrix} f_1'(x) \\ f_2'(x) \end{bmatrix}$$

8th Sep

A = Alice (without replacement)

B = Bob

2 r
3 b

① $P(A = r \text{ and } B = b)$

$= P(A = r) \cdot P(B = b / A = r)$

$= \frac{2}{5} \cdot \frac{3}{4} = \frac{3}{10}$

1 r
3 b

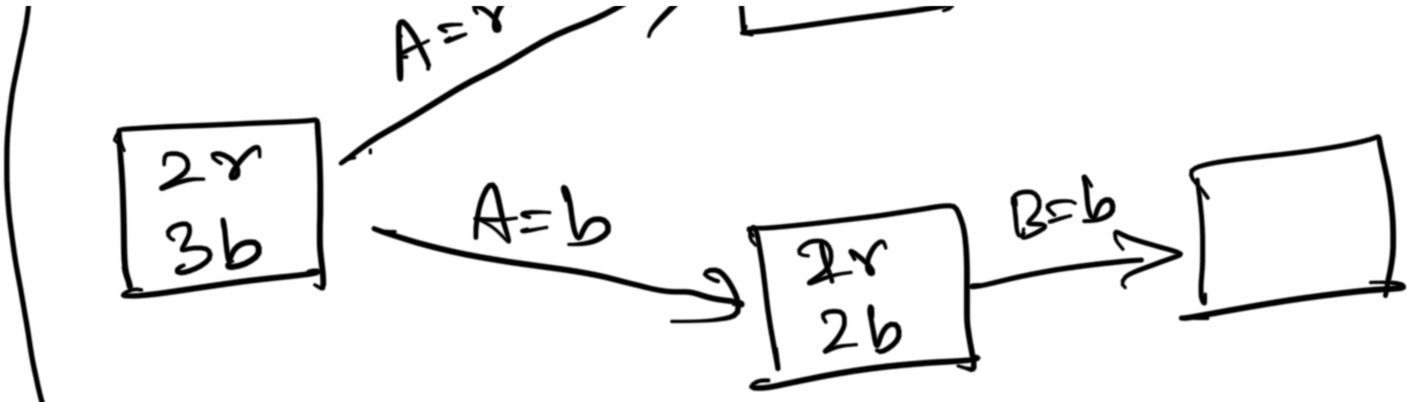
② $P(A = b / B = b)$

$$= \frac{P(B=b/A=b) \cdot P(A=b)}{P(B=b)}$$

$$= \frac{P(B=b/A=b) \cdot P(A=b)}{P(B=b \mid A=r) \cdot P(A=r) + P(B=b/A=b) \times P(A=b)}$$

1r
3b

$\xrightarrow{B=b}$



$$= \frac{\frac{\cancel{3}}{5} * \frac{2}{4}}{\frac{\cancel{3}}{10} + \frac{\cancel{3}}{5} \times \frac{2}{\cancel{4}}}$$

$$= \frac{\frac{1}{10}}{\frac{1}{10} + \frac{1}{10}} = \boxed{\frac{1}{2}}$$

Numerator Layout of Vector Gradients

	$1 \; \boxed{x}$	$n \; \boxed{\bar{x}}$
$1 \; \boxed{f}$	1×1: $\boxed{\frac{df}{dx}}$	$1 \times n$: $\boxed{df/d\bar{x}}$
$m \; \boxed{\bar{f}}$	$m \times 1$: $\boxed{\frac{d\bar{f}}{dx}}$	$m \times n$: $\boxed{\frac{d\bar{f}}{d\bar{x}}}$

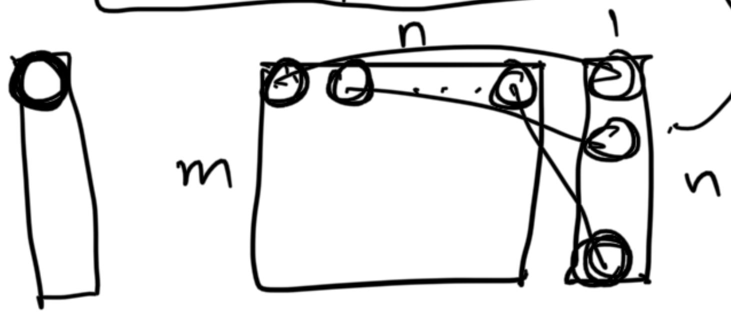
$\bar{y} = A\bar{x} \in \mathbb{R}^m$ $\quad y \in \mathbb{R}^m \quad x \in \mathbb{R}^n$

$A \in \mathbb{R}^{m \times n}$

$A \in \mathbb{R}$ $m=1$

$$\boxed{\frac{d\bar{y}}{d\bar{x}} = A}$$

$$\boxed{y_i = \sum_{k=1}^{m} a_{ik} \cdot x_k}$$



$$\frac{\partial y_i}{\partial x_j} = a_{ij}$$

$$\frac{dy}{dx} = A$$

$$\frac{\partial y_i}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\sum_{k=1}^{m} a_{ik} x_k\right)$$

$$= \sum_{\substack{k=1 \\ k \neq j}}^{m} a_{ik} \underbrace{\cancel{x_k}}_{0} + a_{ij} \underbrace{\cancel{x_j}}_{1}$$

$$\alpha = y^T A x$$

$y \in \mathbb{R}^{(m,1)}$ $x \in \mathbb{R}^{(n,1)}$

$A = m \times n$

$\alpha \to$ scalar

$$\frac{\partial \alpha}{\partial x} = y^T A \qquad \frac{\partial \alpha}{\partial y} = x^T A^T$$

$$\alpha = \alpha^T = (y^T A x)^T = x^T A^T y$$

$$\alpha = x^T A x$$

$A \in \mathbb{R}^{n \times n}$

$x \in \mathbb{R}^n$

$$\frac{\partial \alpha}{\partial x} = x^T (A + A^T)$$

$$\alpha = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j$$

$$\frac{\partial \alpha}{\partial x} = \sum_{i=1}^{n} \frac{\partial \alpha}{\partial x_i} \sum_{j=1}^{n} a_{ij} x_i x_j$$

$$\frac{\partial \alpha}{\partial x_k} = \sum_{i=1}^{n} a_{ik} x_i + \sum_{j=1}^{n} a_{kj} x_j$$

$$= x^T A + x^T A^T$$