$$f(x) = \frac{1}{1+e^{-x}} \rightarrow \underline{\text{Sigmoid}}$$

$\hookrightarrow$ Logistic Regression

$$\frac{d\,(f(x))}{dx} = \,?$$

$\underline{\underline{\text{Sol}^n}}$:

$$\frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{v(x)\cdot u'(x) - u(x)\cdot v'(x)}{[v(x)]^2} \quad (1)$$

$$f(x) = \frac{e^x}{e^x+1}$$

$$\boxed{\frac{d}{dx}(e^x) = e^x}$$

$$\frac{d}{dx}(f(x)) = \frac{(e^x+1)(e^x) - e^x(e^x)}{(e^x+1)^2}$$

$$= \frac{e^x}{(e^x+1)^2} = \boxed{\frac{e^x}{(e^x+1)}}\cdot\frac{1}{(e^x+1)}$$

$$= f(x)\cdot\frac{1}{(e^x+1)}$$

$$\boxed{\frac{d\,(f(x))}{dx} = f(x)\cdot(1-f(x))}$$

---

(1) $\boxed{f(x) = e^x \quad \text{input} = x \in \mathbb{R} \quad \text{output} = f(x) \in \mathbb{R}}$

(2) $\boxed{f(\bar{x}) = 3x^2y \quad \bar{x} \in \mathbb{R}^n \quad f(\bar{x}) \in \mathbb{R}}$ $\bar{x} = (x, y)$

$n=2$

$[\,\ldots\,]\;[\,\ldots\,]\;x \in \mathbb{R}$

③ $\bar{f}(x) = \begin{bmatrix} 2x \\ 3x^2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$ $\bar{f}(x) \in \mathbb{R}^n$ $n=2$

④ $\bar{f}(\bar{x}) = \begin{bmatrix} 3x^2 y \\ 2x+5y \\ \ldots \end{bmatrix}$ $\bar{x} = (x, y)$

$\bar{x} \in \mathbb{R}^n$

$\bar{f}(\bar{x}) \in \mathbb{R}^n$

$\boxed{y = \underline{W}\,\underline{x} + \underline{b}}$

---

② $f(\bar{x}) \in \mathbb{R}$ $\bar{x} \in \mathbb{R}^n$

$\bar{x} = [x_1, x_2, \ldots, x_n]^T$

$$\frac{d}{dx}(f(\bar{x})) = \left[\frac{\partial}{\partial x_1} f(\bar{x}) \quad \frac{\partial}{\partial x_2} f(\bar{x}) \quad \cdots \quad \frac{\partial}{\partial x_n} f(x) \right]$$

④ $\bar{f}(\bar{x}) \in \mathbb{R}^m$ $\bar{x} \in \mathbb{R}^n$

$$\frac{d}{dx}(\bar{f}(\bar{x})) = \begin{bmatrix} \frac{d}{dx}[f_1(\bar{x})] \\ \frac{d}{dx}[f_2(\bar{x})] \\ \vdots \\ \frac{d}{dx}[f_m(\bar{x})] \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial}{\partial x_1} f_1(\bar{x}) & \frac{\partial}{\partial x_2} f_1(\bar{x}) & \cdots & \frac{\partial}{\partial x_n} f_1(\bar{x}) \\ \frac{\partial}{\partial x_1} f_2(\bar{x}) & & & \vdots \\ \vdots & & & \\ \frac{\partial}{\partial x_1} f_m(\bar{x}) & \cdots & \cdots & \frac{\partial}{\partial x_n} f_m(\bar{x}) \end{bmatrix}$$

$$\frac{\partial}{\partial x_1} f_m(x)$$

$$\boxed{\text{Jacobian Matrix}}$$

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} \qquad f'(x) = \begin{bmatrix} f_1'(x) \\ f_2'(x) \end{bmatrix}$$