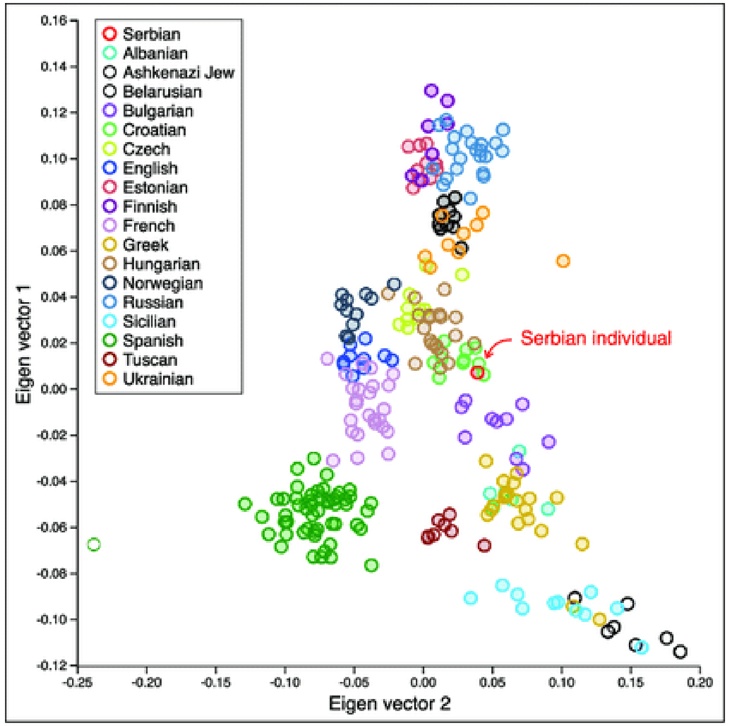
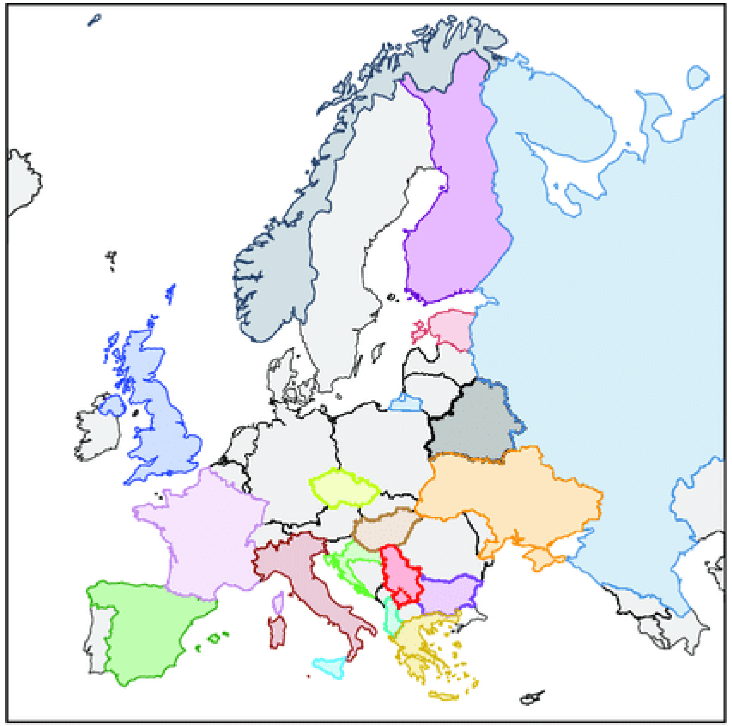
PCA visualization

- Original data
  - One example x for each person in dataset
  - Each person represented by 600,000-dimensional vector of different genetic variants
- PCA
  - Computes Σ
  - v_1 = eigenvector of Σ with largest eigenvalue
  - v_2 = eigenvector of Σ with second largest eigenvalue
- Plot
  - y-axis shows $v_1^T x$ for each x ("first principal component")
  - x-axis shows $v_2^T x$ for each x ("second principal component")



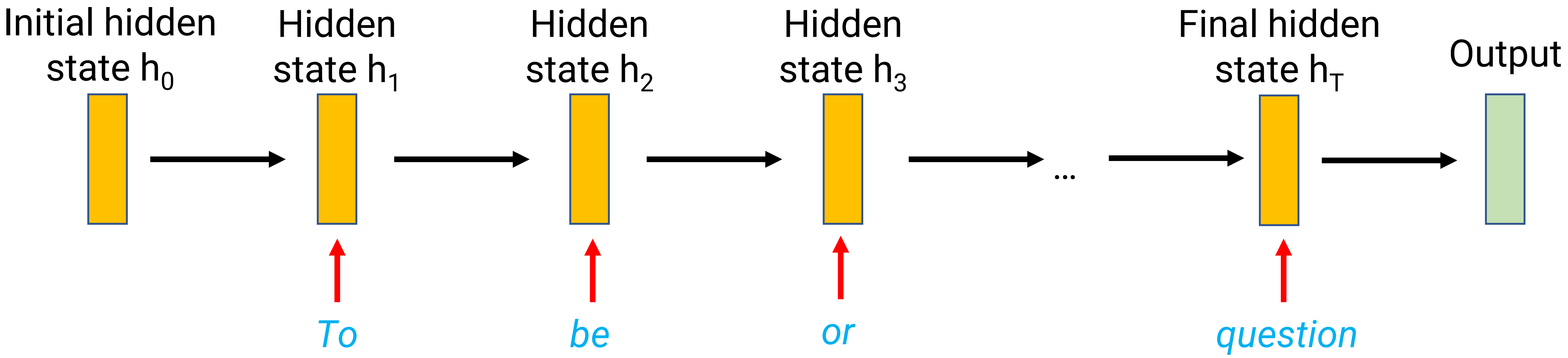


Word Vectors & word2vec

Robin Jia
USC CSCI 467, Fall 2023
November 7, 2023

With a lot borrowed from Jurafsky & Martin, "Speech and Language Processing"
https://web.stanford.edu/~jurafsky/slp3/

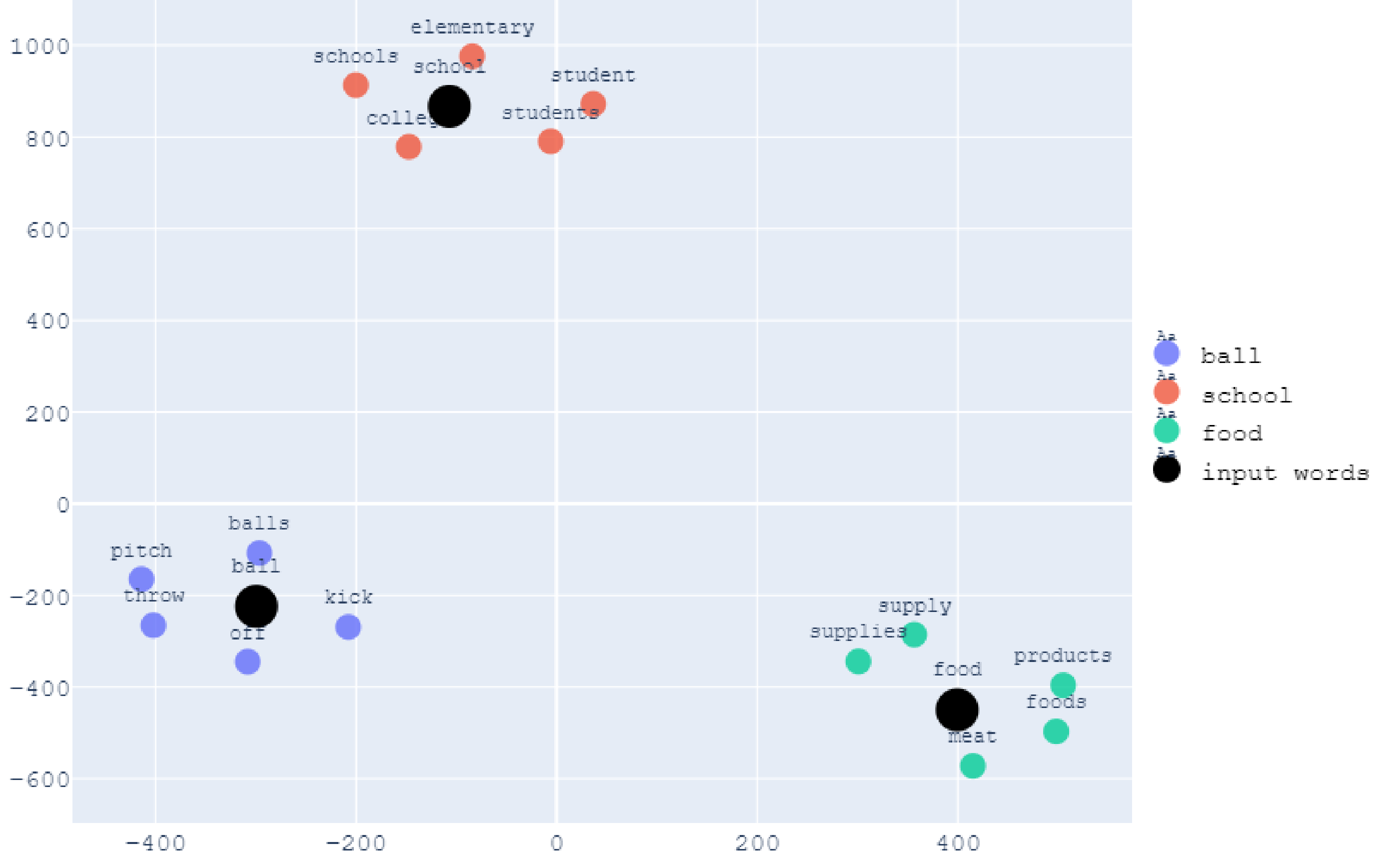
Previously: RNNs



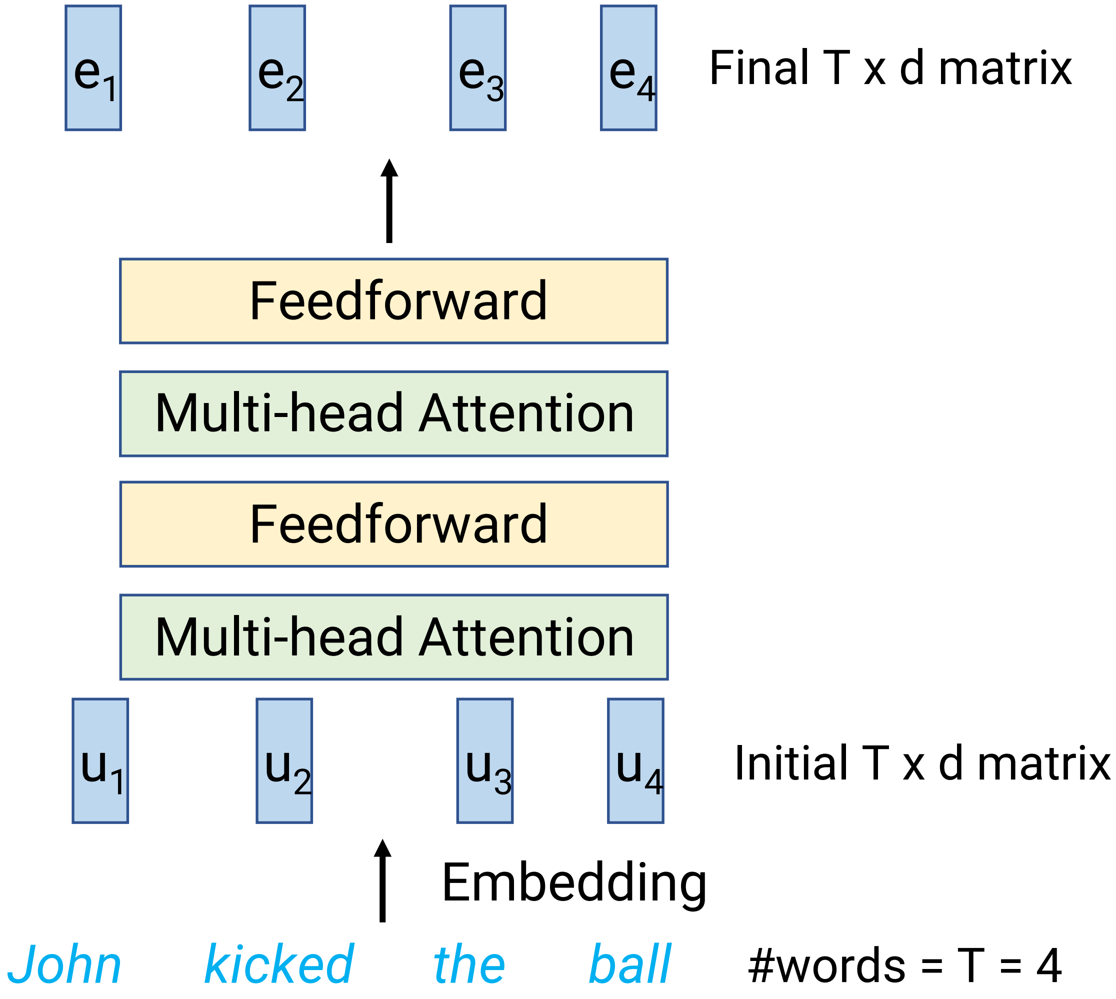
- Idea: Recurrence!
  - "Read" the input one word at a time
  - At each step, update the hidden state of the network
  - Model parameters to do this update are same for each step

Previously: Word Vectors in RNNs

- How do we "feed" the next word to the RNN?
- Want to learn a vector that represents each word
  - For each word *w* in vocabulary *V*, have vector v_w of size *d*
  - |V| * d parameters needed
- Intuition: Similar words get similar vectors



Previously: Transformers



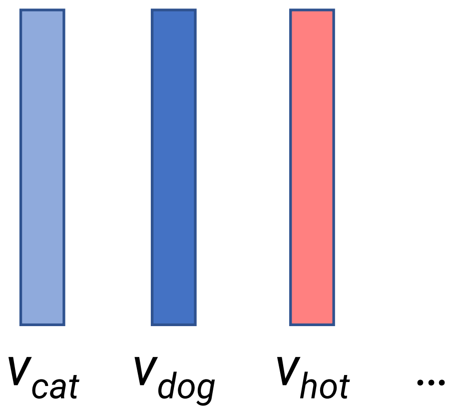
- One transformer consists of
  - **Embeddings for each token of size d**
    - Let T =#tokens, so initially T x d matrix
  - Alternating layers of
    - “Multi-headed” attention layer
    - Feedforward layer
    - Both take in T x d matrix and output a new T x d matrix
  - Plus some bells and whistles
    - Residual connections & LayerNorm
    - Byte pair encoding tokenization

Today: Unsupervised word vectors

- What do we want?
- word2vec
- Solving analogies
- Bias in word vectors

Lexical Semantics

- Goal: For each word *w*, have vector v_w that represents word's meaning
  - Lexical = word-level
  - Semantics = meaning
- What do we want to represent?
  - Synonymy (*car*/*automobile*) or antonymy (*cold*/*hot*)
  - Hypernymy/Hyponymy (*animal*/*dog*)
  - Similarity (*cat*/*dog*, *coffee*/*cup*, *waiter*/*menu*)
  - Various features
    - Sentiment (positive/negative)
    - Formality
    - All sorts of properties (Is a city? Is an action that a person can do?)



The Distributional Hypothesis

- You hear a new word, ***ongchoi***
  - ***Ongchoi*** *is delicious sauteed with garlic.*
  - ***Ongchoi*** *is superb over rice.*
  - *...****ongchoi*** *leaves with salty sauces...*

- Compare with similar contexts:
  - *...****spinach*** *sauteed with garlic over rice...*
  - *...****chard*** *stems and leaves are delicious...*
  - *...****collard greens*** *and other salty leafy greens*

- Conclusion: **ongchoi** is probably a leafy green similar to spinach, chard, and collard greens
- <u>Distributional Hypothesis</u>: Words appearing in similar contexts have similar meanings!
- Firth 1957: “You Shall Know a Word by the Company It Keeps”

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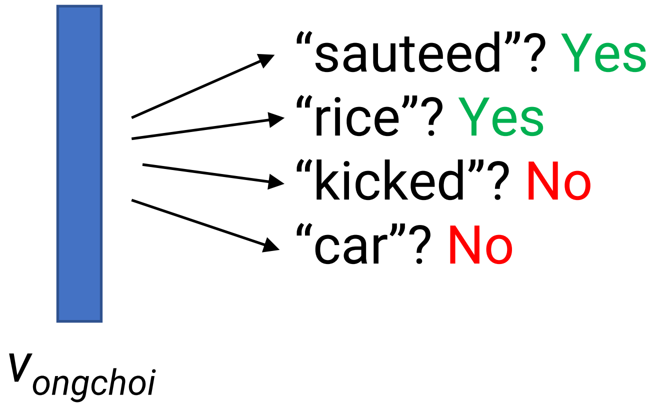
Word vectors as a learning problem

- Want to learn vector v_w for each word *w*
- What makes a vector good?
- Idea: v_w should help you predict which words co-occur with w
  - Captures **distribution** of context words for *w*
  - Think of it as N binary classification problems, where N is size of vocabulary



Creating a dataset

- Given: Raw dataset of text (unsupervised)
- We will create N "fake" supervised learning problems!
  - We don't really care about these supervised learning problems
  - We just care that we learn good vectors
- Task i: Did word *w* co-occur with the i-th word?
  - Positive examples: Real co-occurrences within sliding window
  - Negative examples: Random samples

Word w

… lemon, a tablespoon of apricot jam, a pinch…

Window of radius 2

Word *w* ("input")	**Context *w'* ("task")**	**y (label)**
apricot	tablespoon	+1
apricot	of	+1
apricot	jam	+1
apricot	a	+1

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apricot	jam	+1
apricot	a	+1
apricot	seven	-1
apricot	forever	-1
apricot	dear	-1
apricot	if	-1

How to sample negatives?

- Choose a fixed ratio of negative:positive (e.g. 2)
- Baseline: Sample according to frequency of word p(w) in the data
  - Not ideal because very common words (“the”) get sampled a lot
- Improvement: Sample according to α-weighted frequency

$$p_\alpha(w) = \frac{\text{count}(w)^\alpha}{\sum_{w' \in V} \text{count}(w')^\alpha}$$

  - For α < 1, high-frequency words get down-weighted
  - Typically choose around α=.75

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word2vec model

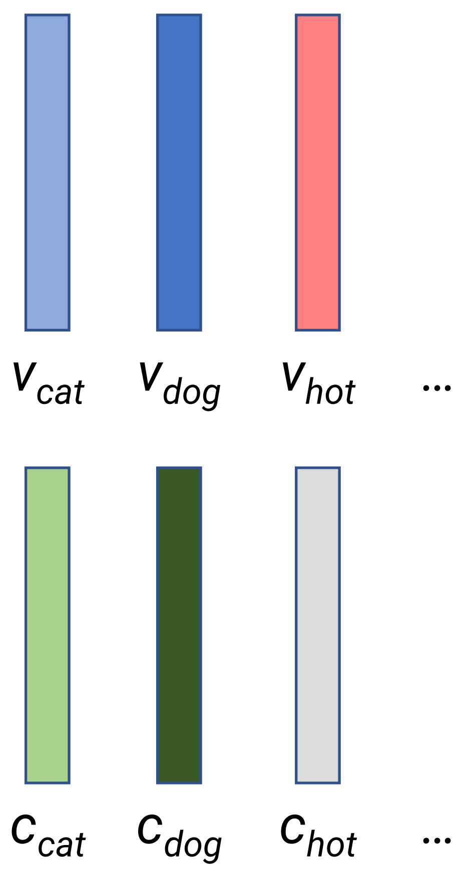
- Parameters (all of dimension d):
  - Word vector v_w for each word ("features"—the actual word vectors)
  - Context vector c_w for each word ("classifier weights" for task corresponding to w as context)
- Goal: v_w can be used by **linear classifier** to do **any** of the N "was this a context word" tasks
- Objective looks just like logistic regression:

$$L(v, c) = \sum_{(w, w', y)} -\log \sigma(y \cdot v_w^\top c_{w'})$$

word context

"features" for word

"weight" for context



Training word2vec

- Strategy: Gradient descent
- Gradient updates essentially same as logistic regression
  - Gradient w.r.t. *c* holds *v* fixed, so it's like *v* are fixed features

$$\nabla_{c_u} L(v,c) = \sum_{(w,w',y):w'=u} -\sigma(y \cdot v_w^\top c_u) \cdot y \cdot v_w$$

Examples where w' = u

Same as logistic regression where v_w is the input x

  - Gradient w.r.t. *v* is symmetrical

$$\nabla_{v_u} L(v,c) = \sum_{(w,w',y):w=u} -\sigma(y \cdot v_u^\top c_{w'}) \cdot y \cdot c_{w'}$$

Examples where w = u

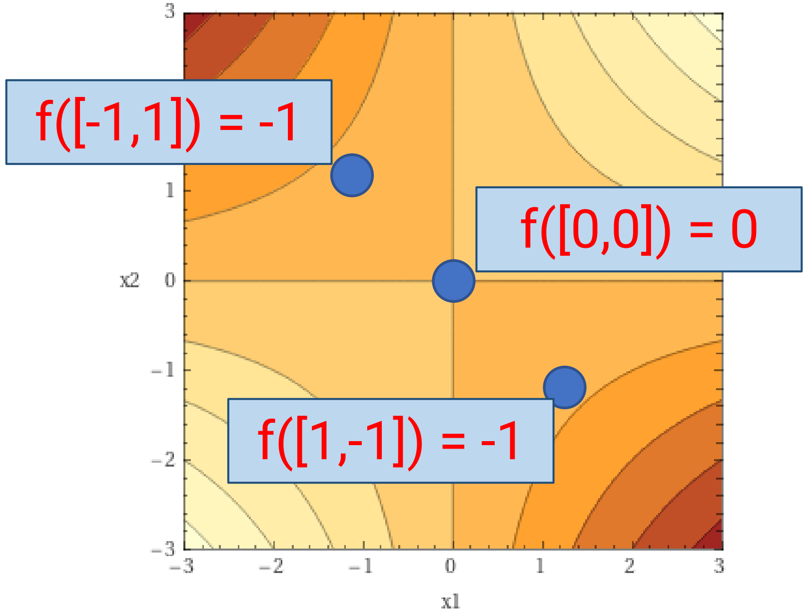
Same as logistic regression where $c_{w'}$ is the input x

Is this a convex problem?

- Looks a lot like logistic regression…
- But it's not convex!
- Why?
  - In logistic regression, we only optimize w.r.t. weights, features are constant
  - Now we optimize both at the same time!
- Fact to remember: f(x) = $x_1 * x_2$ is not convex
  - Consider points [-1, 1] and [1, -1]
  - f(x) = -1 at both points
  - But at the midpoint [0, 0], f(x) = 0
- Corollary: We need to randomly initialize
  - Must break symmetry, as in neural networks

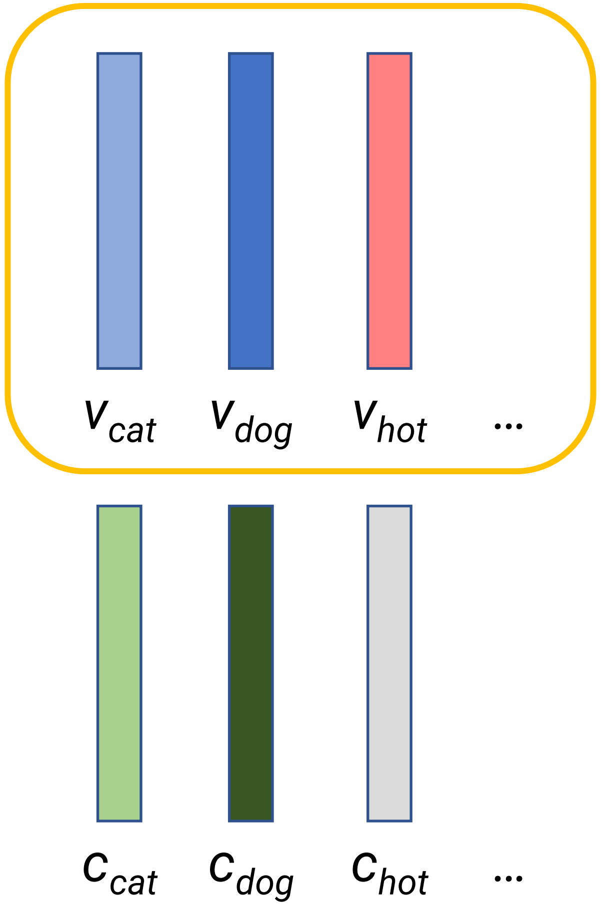
$$L(v, c) = \sum_{(w,w',y)} -\log \sigma(y \cdot v_w^\top c_{w'})$$

Both are optimization variables



word2vec overview

- Acquire large unsupervised text corpus
- Create positive examples for every word by using sliding window
- Create negative examples by randomly sampling context word from weighted word frequency
- Randomly initialize all *v* and *c* vectors
- Train on logistic regression-like loss with gradient descent
- Return *v* vectors
  - c vectors not needed—just helpers



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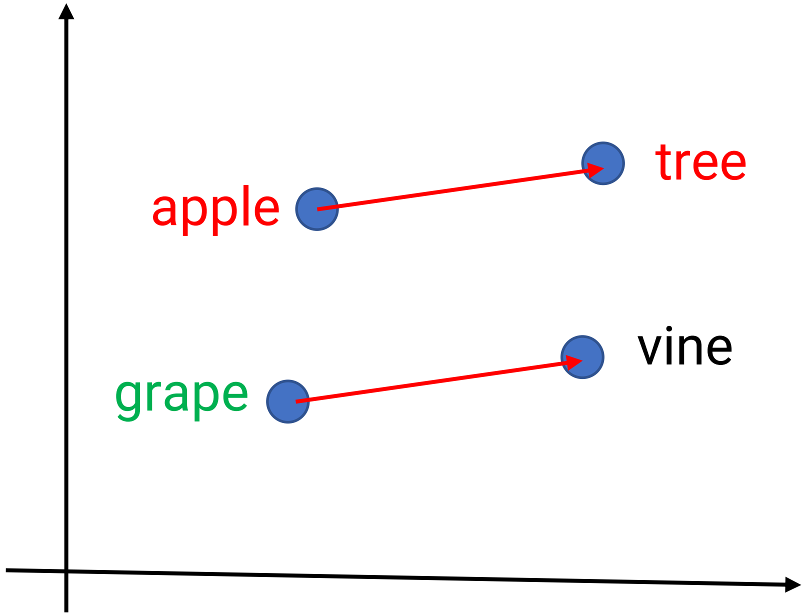
Analogies in vector space

- *Apple is to tree as grape is to…*
- In vector space, resembles a parallelogram
  - Same relationship between apple and tree holds between grape and vine
- $v_{vine} \approx \underbrace{v_{tree} - v_{apple}}_{\text{Represents the “grows on” relation}} + \underbrace{v_{grape}}_{\text{Query word}}$



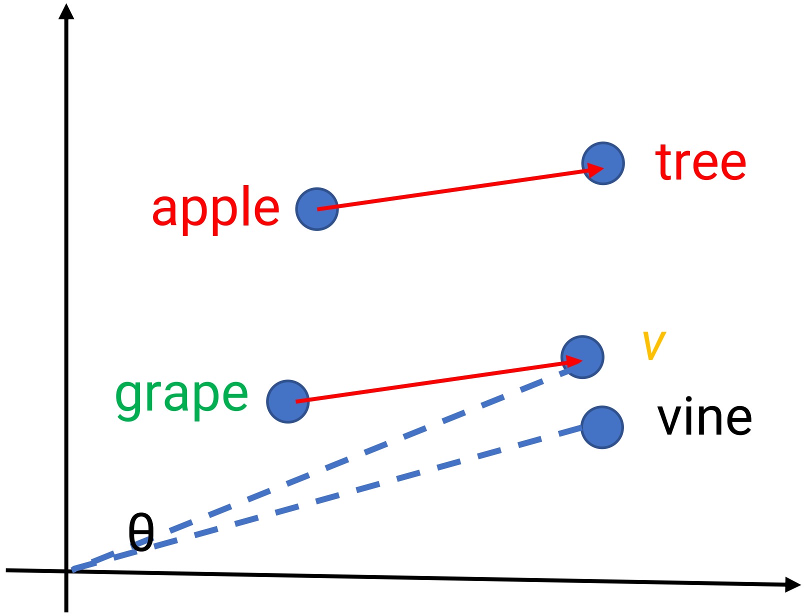
Answering analogy queries

- Compute $v = v_{tree} - v_{apple} + v_{grape}$
- Find word w in vocabulary whose v_w is most similar to v
  - Common choice: Cosine similarity

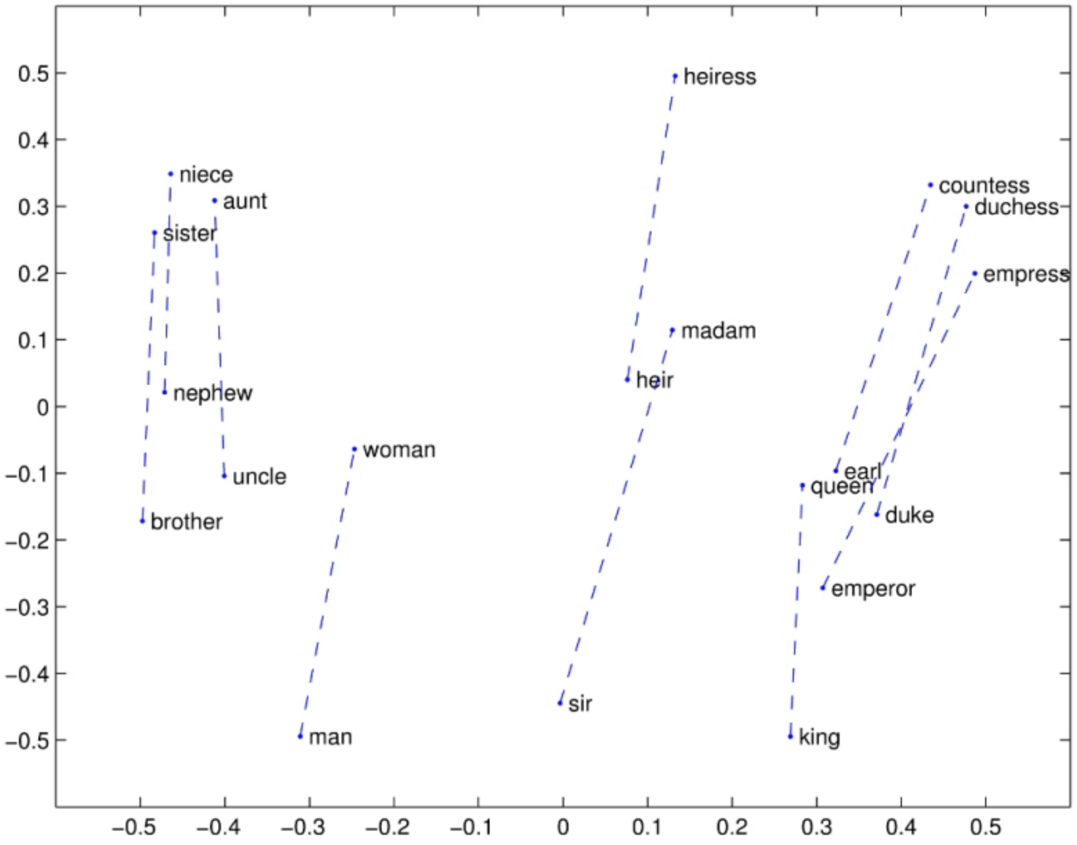
$$\mathrm{cossim}(x, y) = \frac{x^\top y}{\|x\|\|y\|}$$

(= cosine of angle between *x* and *y*)

  - Typically need to exclude words very similar to the query word (e.g. "*grapes*")

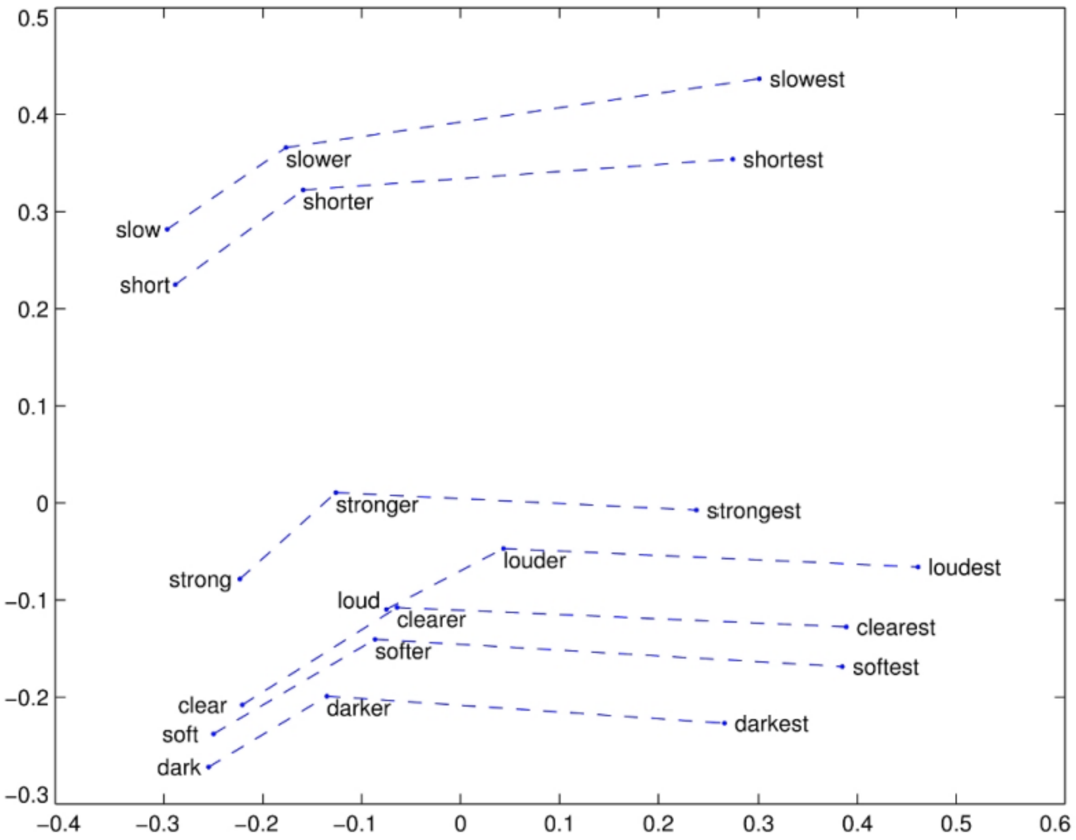


Visualizing Analogies



- Figure: Dimensionality reduction to 2D, then plot words with known relationship
- Roughly same difference between male/female versions of the same word

Visualizing Analogies



- Figure: Dimensionality reduction to 2D, then plot words with known relationship
- Roughly same difference between base, comparative, and superlative forms of adjectives

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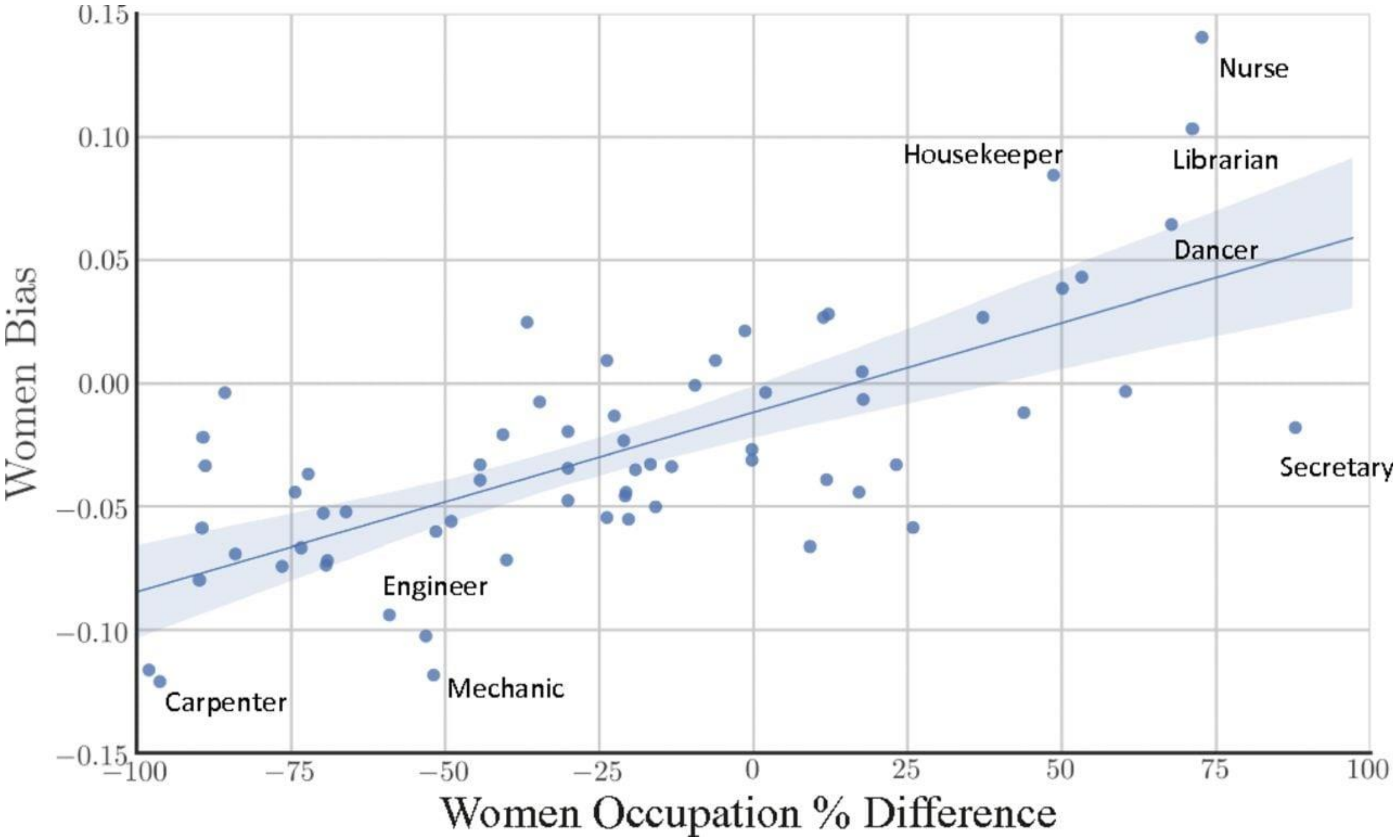
Machine learning is a tornado

- …it picks up everything in its path
- Data has all sorts of associations we may not want to model

What word associations are out there?

- What is *programmer* – *man* + *woman*?
  - According to word vectors trained on news data, it's *homemaker*
  - Existing data has tons of correlations between occupation and gender
- word2vec doesn't know what is a semantic relationship and what is a historical correlation
  - "*queen*" is more related to "*she*" than "*he*" semantically
  - "*nurse*" may co-occur more with "*she*" than "*he*" in available data but not a semantic relationship!

Word vectors quantify gender stereotypes



- X-axis: Real percentage difference in workforce between women & men
- Y-axis: Embedding bias = difference of distance from male-related words and female-related words
- Strong correlation!

Conclusion

- Distributional hypothesis: Words that appear in similar contexts have similar meanings
- word2vec: Learn vectors by inventing a prediction problem (did this word-context pair really occur in the text?)
- Vector arithmetic lets us complete relations
- Vectors capture both lexical semantics and historical biases